New Valid Opperations Enough for the Conjecture of Beal

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Abstract

Theory of numbers is full of operations not permitted for inequations in modular arithmetic. Those walls make impossible to determine solution of some maze expressions. In this document a way to demonstrate one of the most difficult Open Theorems will show how some of those walls can be trespassed.

1. Introduction

As it was mentioned in [1], in the margin of his copy of the works of Diophantus, next to a problem on Pythagorean triples, Pierre de Fermat (1601 – 1665) wrote:

Cubum autem in duos cubos, out quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos ejusdem nominisfas est dividere : cujus rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

(It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain.)

This proposition nowadays is called Fermat's Last Theorem, and it can be expressed like in 1.1.

$$\begin{gathered} \forall a, b, c, n \in \mathbb{N} \\ \textit{where} \\ n > 2 \\ \textit{then} \\ a^n \neq b^n + c^n \end{gathered} \tag{1.1}$$

In this document a new algebra opperation will be presented to solve that famous demonstration in the space of *two margins*. But to get something like that it is necessary to prepare the numbers to corroborate that those forbidden opperations are possible in this case. A new way of demonstrating formulas will be presented: looking for a refutation, every reduction which would invalidate the steps had to be exposed too. The final result would be a brief demonstration, with some other little notes of explanations.

2. The conjecture of Beal

As it is mentioned in [2], the conjecture of Beal is: Let a, b, c, x, y, and z be positive integers with x, y, z > 2. If $a^x = b^y + c^z$, then a, b, and c have a common factor. Or, slightly restated:

$$\begin{gathered}\forall a, b, c, x, y, z \in \mathbb{N} \\ where \\ x>2, y>2, z>2 \\ a, b, c \; coprimes \\ then \\ a^x \neq b^y + c^z\end{gathered} \tag{2.1}$$

So, considering to find refutation, this is the expression needed:

$$\begin{gathered}\forall a, b, c, x, y, z \in \mathbb{N} \\ where \\ x>2, y>2, z>2 \\ a, b, c \; coprimes \\ then \; suppose: \\ a^x = b^y + c^z\end{gathered} \tag{2.2}$$

3. Materials and Methods

For this essay is necessary to work with inequations. More specifically, it is necessary to find the transitive property of the relation of $\neq$.

For example, it is possible to work with the next rule:

$$\begin{gathered}\forall A, B, C, D \in \mathbb{Z} \\ where \\ A \text{ and } B \; coprimes \\ A \text{ and } C \; coprimes \\ |A|>1, |B|>1, |C|>1 \\ then \\ \begin{vmatrix} A & B \\ C & D \end{vmatrix} \neq 0\end{gathered} \tag{3.1}$$

The lemma 3.1 allows us to get equations like $A \cdot D \neq B \cdot C$ after reading

$$A \neq 0 \pmod{B}$$

and

$$A \neq 0 \pmod{C},$$

independently what value had D, if and only if we were working under 3.1 conditions. Considering Lemma 3.1 is trivially easy to demonstrate, and it is no so worthy for this document, the demonstration is not included.

However the next double lemma 3.2 is interesting to get studied.

<table>
<tr><td>$\forall A, B, C, i_1, i_2, j_1, j_2 \in \mathbb{Z}$
$where$
$A^{i_1} \neq C^{j_1} (mod\ B)$
$A^{i_2} \neq C^{j_2} (mod\ B)$
$A, B, C\ coprimes$
$(i_1 + i_2) \cdot (j_1 + j_2) \neq 0 (mod\ \phi(B))$
$then$
$A^{i_1+i_2} \neq C^{j_1+j_2} (mod\ B)$</td><td>$\forall A, B, C, i_1, i_2, j_1, j_2 \in \mathbb{Z}$
$where$
$A^{i_1} \neq -C^{j_1} (mod\ B)$
$A^{i_2} \neq C^{j_2} (mod\ B)$
$A, B, C\ coprimes$
$(i_1 + i_2) \cdot (j_1 + j_2) \neq 0 (mod\ \phi(B))$
$then$
$A^{i_1+i_2} \neq -C^{j_1+j_2} (mod\ B)$</td><td rowspan="3">(3.2)</td></tr>
<tr><td colspan="2">Demonstrations by deduction:</td></tr>
<tr><td>$A^{i_1} = n_1 \cdot C^{j_1} + m_1 \cdot B$
$\left.\begin{cases} A, C, B\ coprimes \\ A^{i_1} \neq C^{j_1} (mod\ B) \end{cases}\right\} \Leftrightarrow n_1 > 1$
$A^{i_2} = n_2 \cdot C^{j_2} + m_2 \cdot B \ \wedge \ n_2 > 1$
$A^{i_1+i_2} = n_1 \cdot n_2 \cdot C^{j_1+j_2} + m_3 \cdot B$
$n_3 = n_1 \cdot n_2 > 1$
$\square$</td><td>$A^{i_1} = -n_1 \cdot C^{j_1} + m_1 \cdot B$
$\left.\begin{cases} A, C, B\ coprimes \\ A^{i_1} \neq -C^{j_1} (mod\ B) \end{cases}\right\} \Leftrightarrow n_1 > 1$
$A^{i_2} = n_2 \cdot C^{j_2} + m_2 \cdot B \ \wedge \ n_2 > 1$
$A^{i_1+i_2} = -n_1 \cdot n_2 \cdot C^{j_1+j_2} + m_3 \cdot B$
$n_3 = n_1 \cdot n_2 > 1$
$\square$</td></tr>
</table>

4. Results and Discussion

For the demonstration of the conjecture 2.1 a test case will be needed. It is necessary to join 2.2 with $b^{y-2} \neq c^{z-2}$ and $b^{y-2} = c^{z-2}$. Firstly, the main branch 2.2 must be reducted considering the next opperations:

$$\begin{gathered} 3^x = b^{2 \cdot Y} + c^{2 \cdot Z} \\ b^{2 \cdot Y} = -c^{2 \cdot Z} (mod\ 3) \\ b^{2 \cdot Y} = 2 \cdot c^{2 \cdot Z} (mod\ 3) \\ (b^Y)^2 mod\ 3 = 2 \cdot (c^Z)^2 mod\ 3 \\ b^Y = c^Z = 0 (mod\ 3) \\ \blacksquare \end{gathered} \tag{4.1}$$

In 4.1 can be observed that if a is 3, then "y=z=0 (modΦ(3))" if incoherent with b and c coprimes to a=3. So, if a=3 then y and z cannot be even. For other hand, what happen if a is not 3 and values y and z are multiple of Φ(a)? To begin that study two test cases will be included to get used later.

<table>
<tr><td colspan="2">$a^x = b^y + c^z \ \wedge \ a \neq 3$</td></tr>
<tr><td>Suppose:
$y - 1 + 1 = z - 1 + 1 = 0 (mod\ \phi(a))$</td><td>Suppose:
$y - 2 + 1 = z - 2 + 1 = 0 (mod\ \phi(a))$</td></tr>
<tr><td colspan="2">$y = z = 0 (mod\ \phi(a))$
$considering\ a^x = b^y + c^z\ under\ Fermat\ Theorem$
$0 = 1 + 1 = 2 (mod\ \phi(a))$
$\blacksquare$</td></tr>
<tr><td colspan="2">Therefore:
$y \neq z (mod\ \phi(a)) \ \vee \ y \neq 0 (mod\ \phi(a))$</td></tr>
</table>

After understanding all the previous steps, now the next demonstration can be accepted:

$Suppose\ a,b,c\ coprimes$ $a^x=b^y+c^z$ $x,y,z>2$ $y\neq z(mod\ \phi(a))\ \vee\ y\neq 0(mod\ \phi(a))$	
$b^{y-2}=c^{z-2}(mod\ a)$	$b^{y-2}\neq c^{z-2}(mod\ a)$
$b^{y-2}\neq -c^{z-2}(mod\ a)$ $b^y=-c^z(mod\ a)\quad [a^x=b^y+c^z]$ $b^{-2}\neq c^{-2}(mod\ a)$ $c^2\neq b^2(mod\ a)$ $b^2-c^2\neq 0(mod\ a)$ $(b-c)(b+c)\neq 0(mod\ a)$ $b\neq -c(mod\ a)\ \wedge\ b\neq c(mod\ a)$ $\left\{\begin{matrix} b^{y-2}=c^{z-2}mod(a) \\ b\neq c(mod\ a)\end{matrix}\right\}\to b^{y-1}\neq c^{z-1}(mod\ a)$ $\left\{\begin{matrix} b^{y-1}\neq c^{z-1}mod(a) \\ b\neq -c(mod\ a)\end{matrix}\right\}\to b^y\neq -c^z(mod\ a)$ $b^y\neq -c^z(mod\ a)\ \wedge\ b^y=-c^z(mod\ a)$ $\blacksquare$	$b^y=-c^z(mod\ a)\quad [a^x=b^y+c^z]$ $b^{-2}\neq -c^{-2}(mod\ a)$ $b^4\neq c^4(mod\ a)$ $b^4-c^4\neq 0(mod\ a)$ $(b^2-c^2)(b^2+c^2)\neq 0(mod\ a)$ $b^2\neq -c^2(mod\ a)$ $\left\{\begin{matrix} b^{y-2}\neq c^{z-2}mod(a) \\ b^2\neq -c^2(mod\ a)\end{matrix}\right\}\to b^y\neq -c^z(mod\ a)$ $b^y\neq -c^z(mod\ a)\ \wedge\ b^y=-c^z(mod\ a)$ $\blacksquare$

5. Conclusions

As it can be appreciated, this demonstration included a technique which enables to work new effective ways with inequations in modular arithmetic. The reason of why it is worthy is because this demonstration was one of most difficult conjectures in algebra, and even being very famous, nowadays is still open. Only Fermat last theorem, easy solvable from the conjecture of Beal, needs pages and pages of difficult geometric explanations. So there are some other theorems easy to demonstrate waiting for the experts who well studied this document.

6. References

[1] A Report on Wiles' Cambridge Lectures, K. Rubin and A. Silverberg. Bulletin of the American Mathematical Society Volume 31, Number 1, July 1994

[2] "A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem", Daniel Mauldin (1997). Notices of AMS. 44 (11): 1436-1439

7. Disclosure Policy

The author declares that there is no conflict of interest regarding the publication of this paper.